Radar basics in calibration

Highlights, Challenges, Impediments, and Development Commitments

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- General picture of a typical radar system
- Radar propagation model
  - Radar range equation
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- MIMO (angle estimation)

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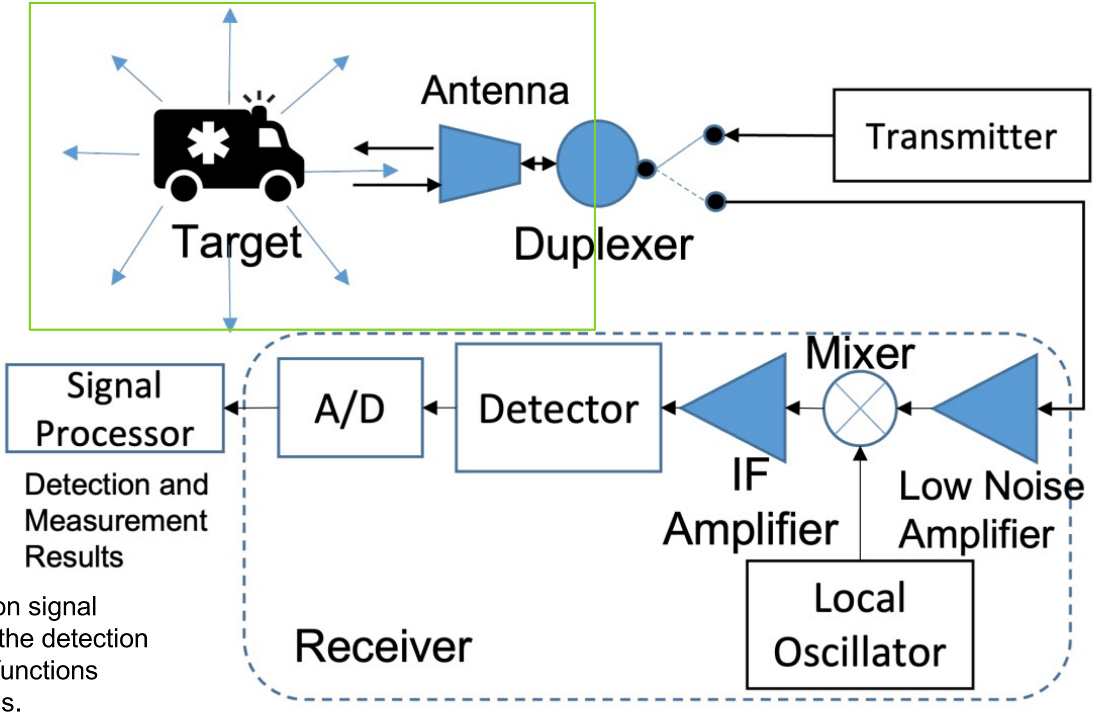
Diagram of typical radar system

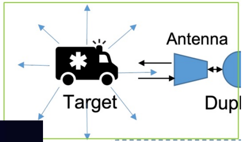
Basic theoretical study on signal properties.

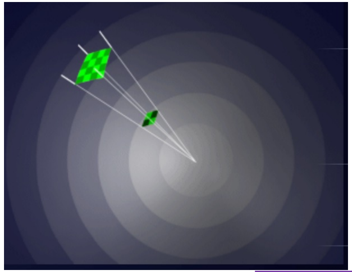


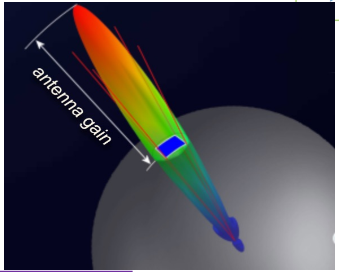
Design of transmitter and receiver various on different types of radar systems according to application.

Algorithm exploration on signal processor, to improve the detection quality or achieve the functions required by applications.

Radar range equation







$$P_r = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 L_{loss}} = \frac{P_t G_t}{4\pi R^2} \cdot \sigma \cdot \frac{1}{4\pi R^2} \cdot \frac{\lambda^2 G_r}{4\pi} \cdot \frac{1}{L_{loss}}$$

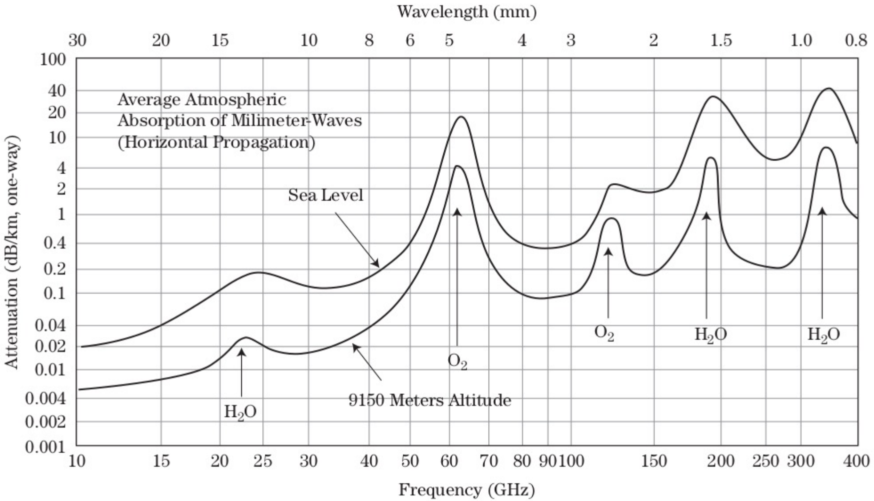
- Power density at the target place;
- Reflected power at the target;
- Reflected power at the destination;
- Effective antenna aperture $A_w = \frac{\lambda^2 G_r}{4\pi}$
- P_t: Transmit power;
- G_t, G_r: Antenna gain of transmitter and receiver;
- λ: Wavelength of radar signal;
- σ: RCS or target;
- R: Range between radar and target;
- L_{loss}: Other propagation losses.

Typical frequency bands of automotive radar: 77GHz, 94GHz, 150GHz, 300 GHz.

The loss factor L_{loss} includes:

$$L_{loss} = L_a L_c L_o L_x$$

- L_a: Atmospheric propagation loss;
- L_c: Loss due to sensor cover material;
- L_o: Loss due to obstructions (E.g. contaminants on sensor cover, rain drops);
- L_x: Loss cause by the other factors in the path.

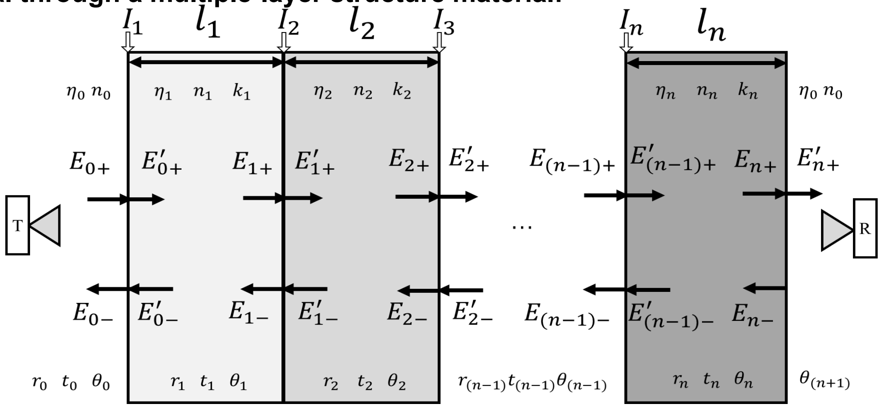


One-way atmospheric attenuation as a function of frequency at sea level and at 9150 meters altitude.

[1] https://www.radartutorial.eu/01.basics/The%20Radar%20Range%20Equation.en.html
[2] https://www.radartutorial.eu/06.antennas/an07.en.html

Theory of transmission and reflection estimation on multi-layer material

Electric field of transmission process and reflection when radar signal through a multiple-layer structure material:



Definitions of parameters:

Parameters	Definitions
E_{i+}, E_{i-}, E'_{i+}	Incident, reflected and transmitted electric field
n_i	Refractive index of medium i
l_i	Thickness of medium i
ω	Angular frequency of radar signal
μ_i	Permeability
δ_i	Conductivity
ε_0	Vacuum permittivity
ε_i	Relative permittivity of the material
θ_i	Incident ray angle

Reflection coefficient

$$r_i = \frac{\eta_{i+1}cos\theta_i - \eta_i cos\theta_{i+1}}{\eta_{i+1}cos\theta_i + \eta_i cos\theta_{i+1}}$$

Transmission coefficient

$$t_i = \frac{2\eta_{i+1}cos\theta_i}{\eta_{i+1}cos\theta_i + \eta_i cos\theta_{i+1}}$$

Snell's law.

Wave impedance: $\eta_i = \sqrt{\frac{j\omega\mu_i}{\delta_i + j\omega\varepsilon_0\varepsilon_i}}$

Wave number: $k_i = \omega\sqrt{\varepsilon_0\varepsilon_i\mu_i}$

Propagation matrix:

$$\begin{bmatrix} E_{0+} \\ E_{0-} \end{bmatrix} = \frac{1}{t_0}\begin{bmatrix} 1 & r_0 \\ r_0 & 1 \end{bmatrix}\begin{bmatrix} e^{jkl_1} & 0 \\ 0 & e^{-jkl_1} \end{bmatrix} \cdots \frac{1}{t_n}\begin{bmatrix} 1 & r_n \\ r_n & 1 \end{bmatrix}\begin{bmatrix} e^{jkl_n} & 0 \\ 0 & e^{-jkl_n} \end{bmatrix}\begin{bmatrix} E'_{n+} \\ 0 \end{bmatrix} = \begin{bmatrix} M_1 & M_3 \\ M_2 & M_4 \end{bmatrix}\begin{bmatrix} E'_{n+} \\ 0 \end{bmatrix}$$

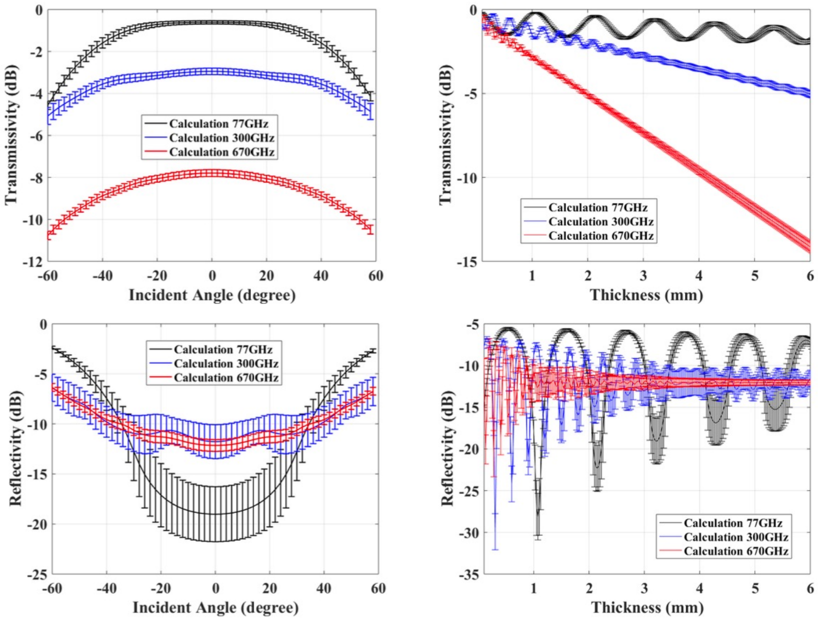
Transmissivity: $T = \left|\frac{E'_{n+}}{E_{0+}}\right|^2 = \left|\frac{1}{M_1}\right|^2$

Reflectivity: $R = \left|\frac{E_{0-}}{E_{0+}}\right|^2 = \left|\frac{M_2}{M_1}\right|^2$

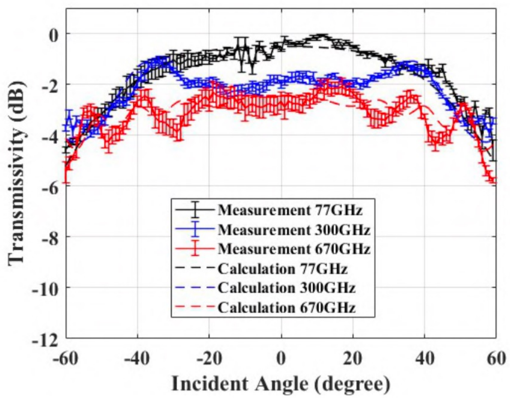
Rain attenuation: F. Norouzian *et al.*, "Rain Attenuation at Millimeter Wave and Low-THz Frequencies," in *IEEE Transactions on Antennas and Propagation*, vol. 68, no. 1, pp. 421-431, Jan. 2020, doi: 10.1109/TAP.2019.2938735.

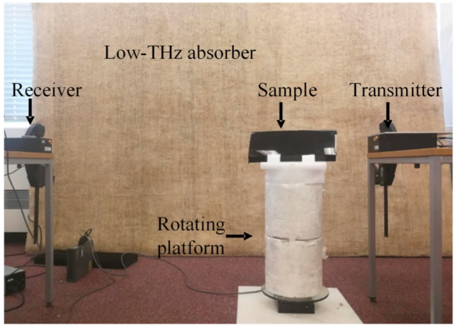
Transmissivity and reflectivity as function of incident angle and thickness of material. (One example of vehicle bumper material)

Computed transmissivity and reflectivity



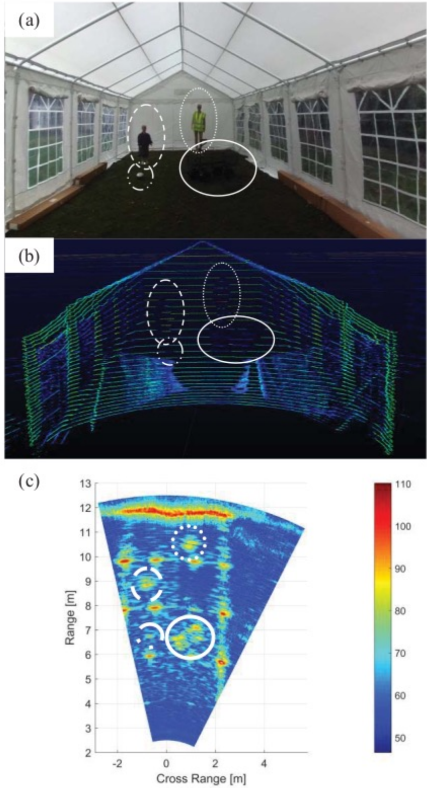
Comparison between computed values and measured values

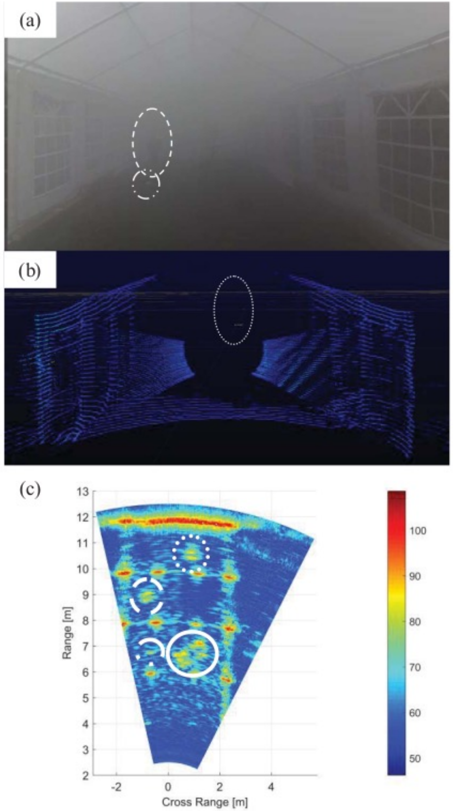


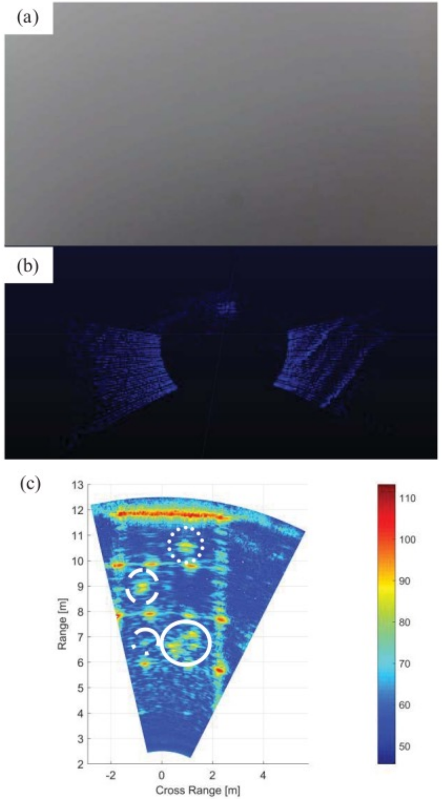


Y. Xiao, F. Norouzian, E. G. Hoare, E. Marchetti, M. Gashinova and M. Cherniakov, "Modeling and Experiment Verification of Transmissivity of Low-THz Radar Signal Through Vehicle Infrastructure," in *IEEE Sensors Journal*, vol. 20, no. 15, pp. 8483-8496, 1 Aug.1, 2020, doi: 10.1109/JSEN.2020.2982984.

Advantage of radar system



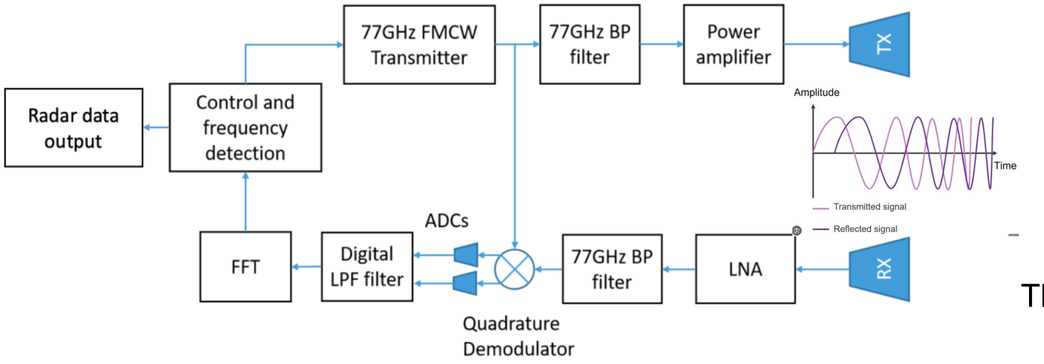


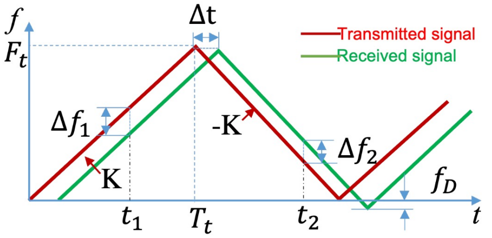


(a) camera; (b) Lidar; (c) radar

L. Daniel, D. Phippen, E. Hoare, A. Stove, M. Cherniakov and M. Gashinova, "Low-THz radar, lidar and optical imaging through artificially generated fog," *International Conference on Radar Systems (Radar 2017)*, Belfast, 2017, pp. 1-4, doi: 10.1049/cp.2017.0369.

FMCW basics (How to get range and velocity information)





Range estimation: $R = \frac{c\Delta t}{2} = \frac{c\Delta f}{2K}$

Velocity estimation:
Doppler frequency shift between up- and down chirps which is caused by target movement:

$$f_D = \frac{\Delta f_1 - \Delta f_2}{2}$$

Velocity of object: $v = \frac{cf_D}{2f_0}$

- Frequency between transmitted and received signal determines range.
- Frequency difference between chirps of received signal determines velocity.

The transmitted signal:

$$S_t = A_t exp\left(j\pi(2f_0t + Kt^2)\right)$$

The received signal:

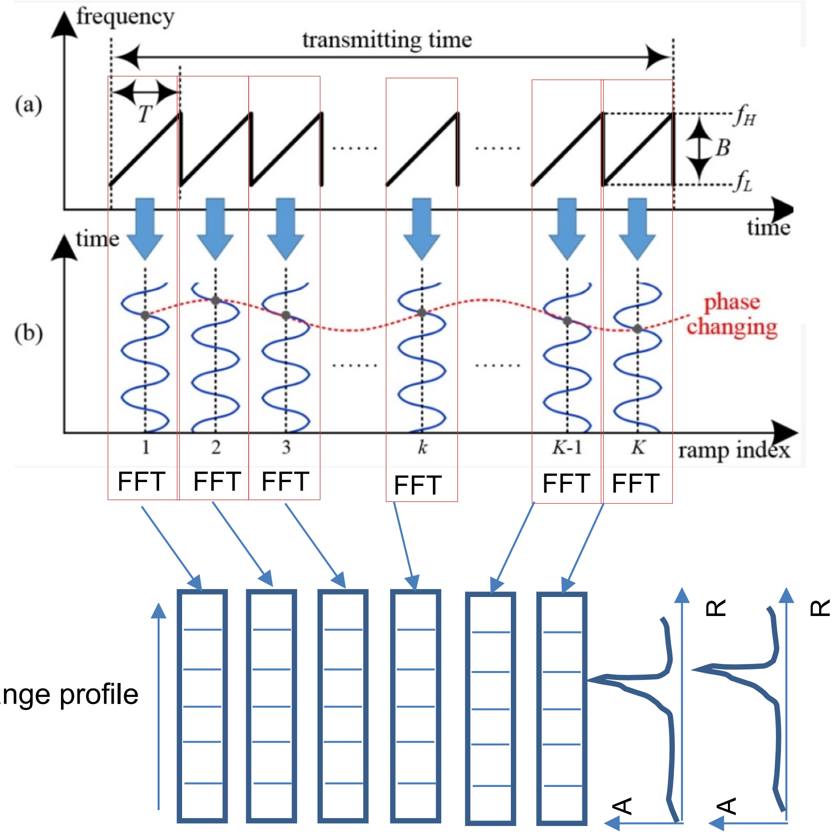
$$S_r = A_r exp\left(j\pi(2f_0(t + \Delta t) + K(t + \Delta t)^2 + \Phi_d)\right)$$

The mixed signal:

$$S_m = A_m exp\left(j\pi(2f_0t + 2K\Delta tt + K\Delta t^2 + \Phi_d)\right)$$

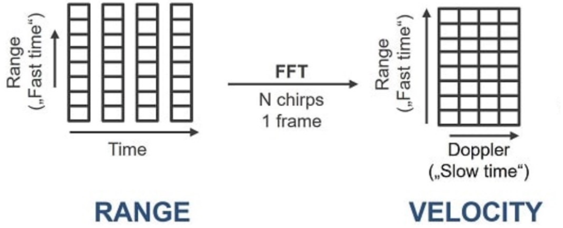
- A_t is the amplitude of transmitted signal;
- A_r is the amplitude of received signal;
- A_m is the amplitude of mixed signal;
- f_0 is the carrier frequency of radar signal;
- K is the slope of modulated signal, $K = \frac{F_t}{T_t}$, where F_t is the bandwidth and T_t is the duration of half chirp (or one chirp, depends on the chirp pattern);
- ϕ_d Doppler phase shift;
- $\Delta f = K\Delta t$ is the intermediate frequency (IF) signal extracted for distance estimation;

2D-FFT (How to get range and velocity in signal processing level)

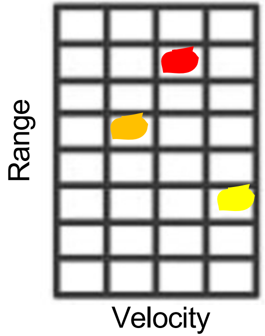


Range profile

Stack the range profile of multiple chirps as a matrix for Doppler FFT.



Range-velocity map:
Grid with higher intensity value indicate an object.



Why ambiguous radial velocity?

- Maximum unambiguous radial velocity: $v_{max} = \frac{c f_{PRF}}{4 f_{tx}}$,
  f_{PRF} is the pulse reptation frequency and f_{tx} is the frequency of transmitted signal..

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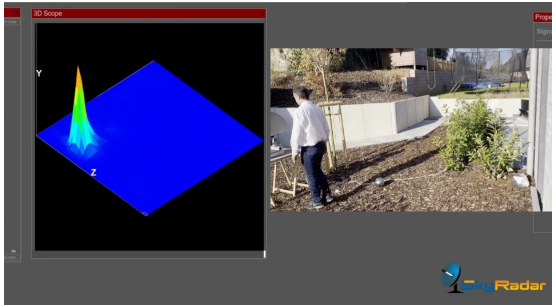


https://www.skyradar.com/blog/video-fft-plot-of-a-pulsed-doppler-radar-implemented-with-skyradars-nextgen-radar-and-freescopes

One example of mechanical scanning imaging radar

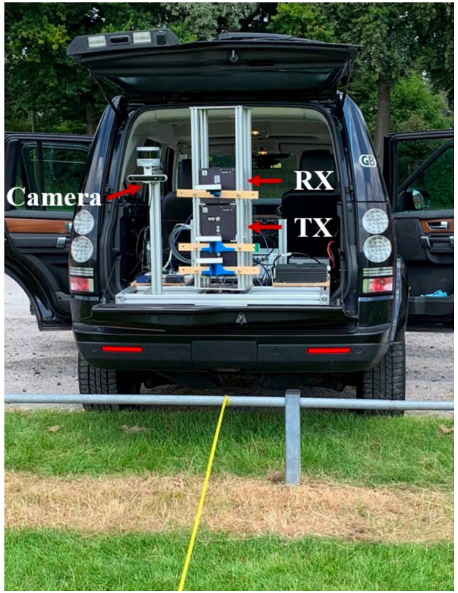
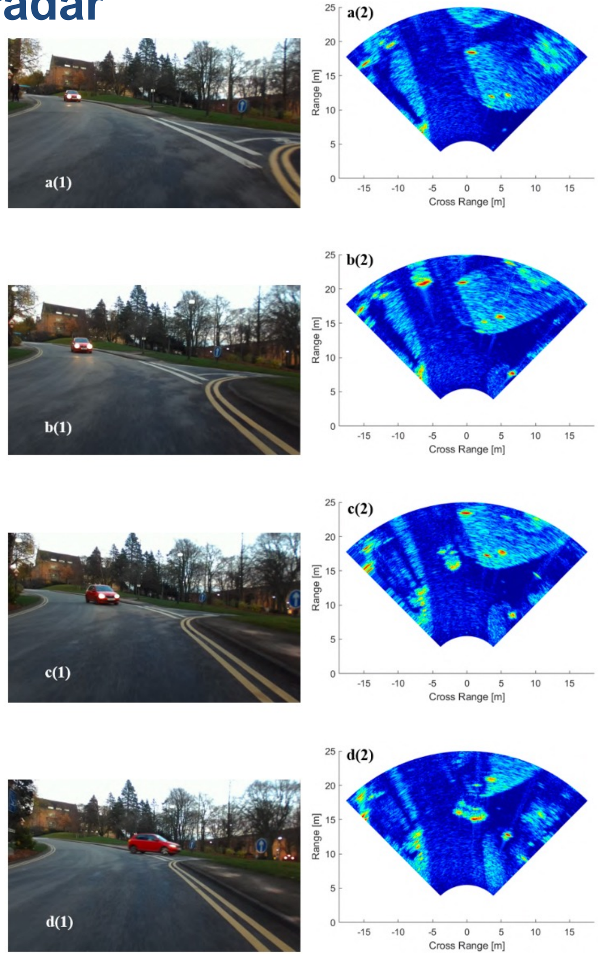


Table 5.1 79 GHz FMCW imaging radar parameters.

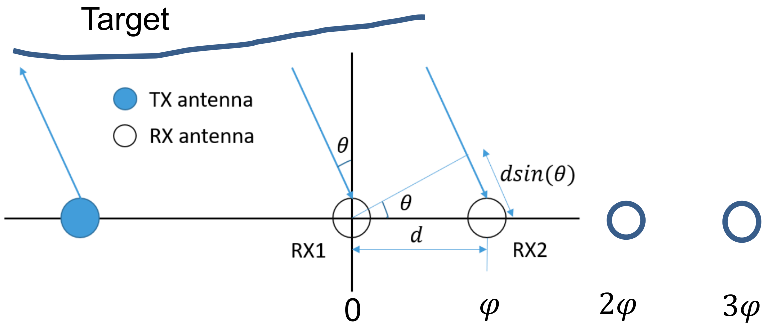
Parameter	Value	Units
Centre Frequency	78.5	GHz
Bandwidth	5	GHz
Transmit Power	13	dBm
PRF/PRI	232/4.3	Hz/ms
Chirp Duration	1	ms
Az. Beam Width (2-way)	1.7	°
El. Beam Width (2-way)	7.2	°
Antenna Gain	30	dBi
Polarization	VV	



- Narrow the azimuth beam width
- Decrease the pulse repetition rate.
- Mechanically changing radiation direction of antenna in a step of around 1 degree.
- Clutters are valuable information.
- Lost doppler information.

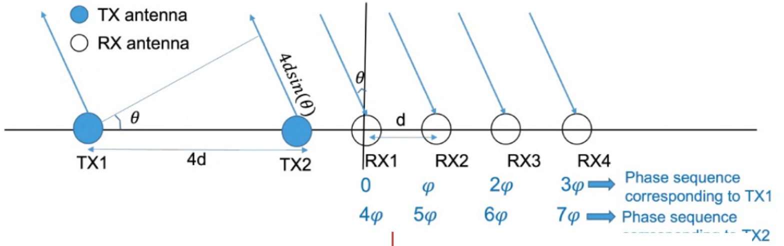
MIMO (Multiple input multiple out -> angle estimation)

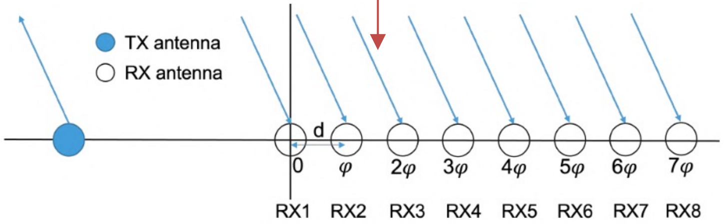
Single input and multiple output (SIMO)

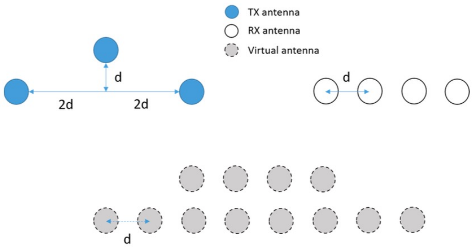


- Signal received by RX2 passes a longer distance of $dsin(\theta)$ than signal to RX1.
- The phase shift between RX1 and RX2 can be denoted as $\varphi = \left(\frac{2\pi}{\lambda}\right) dsin(\theta)$, $\boldsymbol{\lambda}$ is the wavelength.
- The angle of arrival $\theta = sin^{-1}\left(\frac{\varphi\lambda}{2\pi d}\right)$.
- Phase difference can only be uniquely estimated in range of $(-\pi, \pi)$ due to the limitation of hardware. The unambiguous FOV is $\theta_{FOV} = \pm sin^{-1}\left(\frac{\lambda}{2d}\right)$.
- Maximum FOV is $\theta_{FOV} = \pm 90^{\circ}$ with $d = {}^{\lambda}/_{2}$.
- The angular resolution can be improved by increasing the number of RXs, N: $\theta_{res} = \frac{2}{N}$.
- Angle estimation is achieved by estimating frequency across N receiver channels. Increasing the number of receivers actually is improve the sampling rate of angle-FFT.

MIMO

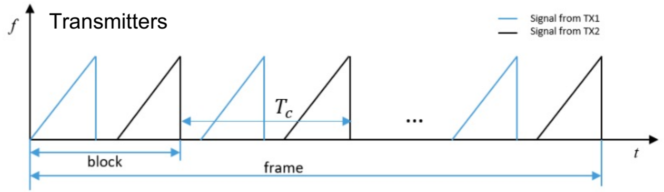


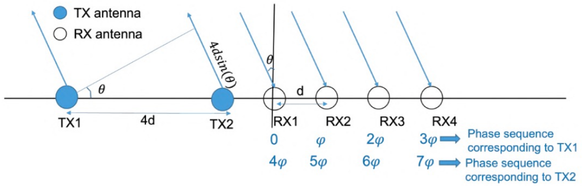


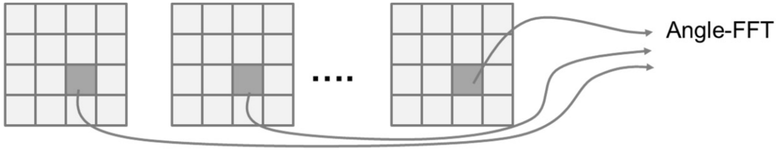


MIMO modulation/demodulation

TDM (Time division multiplexing)-MIMO







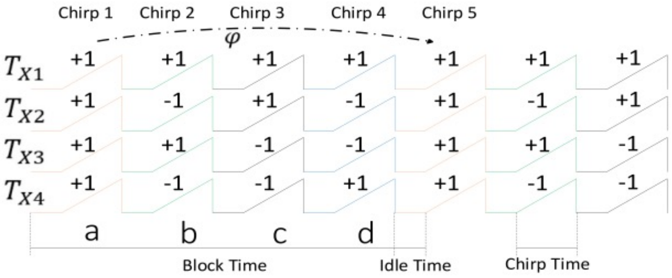
2D-FFT of virtual antenna 1 (TX1 & RX1)

2D-FFT of virtual antenna 2 (TX1 & RX2)

2D-FFT of virtual antenna 8 (TX2 & RX4)

- Only one transmitter is active at any time
- Sum 2D-FFT matrices over virtual antennas
- C-FAR detection algorithm identifies peaks in this matrix that correspond to valid objects.
- For each valid object, angle-FFT is performed on the corresponding peaks to identify the angle of arrival of that object.
- Prior to applying angle-FFT, a Doppler correction step must be performed in order to correct for any velocity induced phase change.

BPM (Binary phase modulation)-MIMO



Binary phase modulation chirp sequences with four transmitters

Decoding according to chirps in one block time $S_{a,b,c,d}$ to obtain virtual signals $S_{1,2,3,4}$.

$$
\begin{aligned}
S_1 &= (S_a + S_b + S_c + S_d)/4 \\
S_2 &= (S_a - S_b + S_c - S_d)/4 \\
S_3 &= (S_a + S_b - S_c - S_d)/4 \\
S_4 &= (S_a - S_b - S_c + S_d)/4.
\end{aligned}
$$

- Modulate the initial phase of chirps over different TXs.
- Phases are either 0º or 180º (equivalent to multiplying each chirp by +1 or –1).
- Simultaneous transmission across multiple TX antennas.
- Demodulate at the receiver side.